LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2014**

MT 1503 - ANALYTICAL GEOMETRY OF 2D, TRIGNOMETRY AND MATRICES

Date :10/11/2014 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

PART – A

Answer ALL questions: **(10 × 2 = 20)**

1. What is the expression for tan nθ in terms of tan θ.
2. How can you group the pairs in the expansion of cos nθ when n is odd and even?
3. Complete the relation:

   $\cosh^2 x + \sinh^2 x =$

   $\cosh^2 x - \sinh^2 x =$
4. Write the expansion of $\cosh^{-1} x$ and $\sinh^{-1} x$.
5. Define skew symmetric matrix and give an example of it.
6. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
7. What is the condition for the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ to be conjugate?
8. Write the pole of the line $ax + by + c = 0$ with respect to the parabola $y^2 = 4ax$.
9. Define rectangular hyperbola.
10. Define polar equation of a conic.

PART – B

Answer any FIVE questions: **(5 × 8 = 40)**

11. Expand Cos 6θ in terms of sin θ.
12. If $\frac{\tan\theta}{\theta} = \frac{2524}{2523}$, find θ approximately.
13. If sin (A + iB) = x + iy, prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ *and* $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.
14. Find the value of Log $\frac{1+\cos\theta+i\sin\theta}{\cos\theta-1+i\sin\theta}$.

15. Find the eigen values of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

16. Find the locus of the poles of all tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$.

17. Find the locus of the midpoints of the chords of the parabola which subtend a right angle at the vertex of the parabola.

18. The asymptotes of a hyperbola are parallel to $2x + 3y = 0$ and $3x - 2y = 0$ its centre is at (1,2) and it passes through the point (5,3). Find its equation and its conjugate.

PART – C

Answer any TWO questions: **(2 × 20 =40)**

19. (a) Prove that $64(\text{Cos}^8\theta + \text{Sin}^8\theta) = \text{Cos } 8\theta + 28 \text{ Cos } 4\theta + 35$.

(b) Prove that $\text{Cos } 8\theta = 1 - 32\text{Sin}^2\theta + 160\text{Sin}^4\theta - 256\text{Cos}^6\theta + 128\text{Sin}^8\theta$.

20. (a) If $\cos\alpha \cosh\beta = \cos\phi \sin\alpha \sinh\beta = \sin\phi$ Prove that $\sin\phi = \pm\sin^2\alpha = \pm\sinh^2\beta$.

(b) Reduce $(\alpha + i\beta)^{x+iy}$ to the form A + iB. (12+8)

21. Diagonalize the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$.

22. (a) Show that the locus of the intersection of tangents to $y^2 = 4ax$ which intercepts a constant length d on the directrix is $(y^2 - 4ax)(x+a)^2 = d^2x^2$.

(b) Trace the curve $\frac{12}{r} = 4 + \sqrt{3}\,Cos\,\theta + Sin\,\theta$.

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